Quiz 6, MATH 240, Fall 2023

Write your name clearly.

Name:

UID:

(1) (5 points) Recall that $\mathbb{P}_4$ is the vector space of polynomials with degree at most 4. Let H be the subset of all polynomials of the form $f(x) = a + x^4$, where a is in $\mathbb{R}$. Give one reason why H is not a subspace of $\mathbb{P}_4$.

For any $f(x)$ in H, the coefficient of x^4 is 1, so the zero polynomial is not in H.

(Note that the zero polynomial in $\mathbb{P}_4$ is
$0 + 0x + 0x^2 + 0x^3 + 0x^4$)

(2) (15 points) Let $A = \begin{pmatrix} 1 & -6 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}$. Write down a set of vectors that spans

      (a) $\mathrm{Col}(A)$;   (b) $\mathrm{Row}(A)$;   (c) $\mathrm{Nul}(A)$.

Note: your set doesn't have to be a basis, just a spanning set.

(a) $\mathrm{Col}(A) = \mathrm{Span}\{(1,0), (-6,0), (0,1), (0,-1)\}$

(b) $\mathrm{Row}(A) = \mathrm{Span}\{(1 \quad -6 \quad 0 \quad 0), [0 \quad 0 \quad 1 \quad -1]\}$

(c) Note that

$$A(x_1, x_2, x_3, x_4) = \vec{0} \iff x_1 = 6x_2,$$
$$x_3 = x_4$$

$$\iff \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = x_2 \begin{pmatrix} 6 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}.$$

So $\mathrm{Nul}(A) = \mathrm{Span}\{(6,1,0,0), (0,0,1,1)\}$.